Microeconomics with Ethics

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Chapter 8
Demand Tools – Demand Elasticities and Consumer Surplus

This chapter develops several tools related to consumer demand and demand functions that will be used throughout the course. The first set of tools discussed are elasticities, which are ways to measure the magnitudes of some of the cause and effect relationships described in Chapter 7. Recall that actual demand functions are rarely, if ever, derived for specific products because it is difficult to know what consumers would do in circumstances that may never before have occurred. However, it is possible to calculate the demand sensitivities to changes in some of the independent variables in the neighborhood of actual values that have occurred. This is the focus of elasticity measures.

The second set of tools we begin to develop in this chapter is a measurement of the surplus value attained by traders in a market when they purchase or sell a good. Surplus measures the increase in well-being, or happiness, of traders by virtue of their purchases of products in the market. Surplus measures will be used to highlight the benefits that accrue to the traders and how benefits change when market variables change. Analyzing changes in surplus is referred to as welfare analysis because it's a way to measure the changes in well-being of consumers in the market. Later we will also develop a surplus measure for producers, who are on the other side of the trades taking place in a market.

8.1 Demand Elasticities

Learning Objectives

1. Learn how elasticities are used to measure to measure the sensitivity of demand to changes in the independent variables.
2. Learn to measure and interpret various types of demand elasticities.

Throughout this course we will be analyzing economic cause and effect relationships, asking questions like, how will market demand change, or later, how will the equilibrium market price and quantity change, when there is an increase in the price of a substitute good. Oftentimes, simply knowing the direction of the cause and effect relationships is adequate to provide a good understanding of a market. However, sometimes we would like to know more. Sometimes we care to know the magnitudes of the changes. Knowing by how much something changes, is a measure of the sensitivity of the dependent variable to changes in the independent variables, and can also help in predicting the numerical values of the final outcomes. This is where elasticities come in.

One way to measure of the sensitivity of a relationship is to consider the slope of that relationship when plotted on a diagram. However, this measure poses a problem because the units of such a measure will vary from product to product since units of measure can differ

dramatically. Some goods will be measured by counting them, others measured in kilograms and others in thousands of tons. Some prices will be in dollars and others in euros, etc.

To resolve this problem, economists have devised a unitless measure of sensitivities called elasticities. There are several elasticity measures we'll introduce in this chapter relating to demand, including the price elasticity of demand, the cross-price elasticity of demand and the income elasticity of demand.

Price Elasticity of Demand

The Price Elasticity of Demand, or PED, measures how much demand for a product changes (measured in percentage terms) given a change in the price (also measured in percentage terms). By using percentages, we solve the problem concerning different units of measure.

The percentage change in a variable, say P, can be found using the following formula.

$$\%\Delta P = (P2 - P1) / P1 = (\Delta P/P)$$

where the greek letter delta (Δ) is synonymous with the phrase "change in," P1 is the initial, or starting, price, and P2 is the ending price. One mnemonic I always use for percentage changes is (New – Old) / Old, where New is the final value of the variable, and Old is the original or starting value of the variable. Note that we can write this in several ways, with the first expression showing the initial and final values explicitly, and the second expression abbreviating the change in price using the greek letter delta. Note that a percentage change is a unitless value, because both the numerator and the denominator of the expression is measured in, say \$/lb, and therefore the ratio cancels the units out.

The price elasticity of demand is defined as the ratio of two percentages changes, with *the % change in the quantity demanded* (the dependent variable) in the numerator *relative to the % change in the price* (the independent variable) in the denominator, as shown below.

$$PED = - (\Delta Q_D/Q_D) / (\Delta P/P)$$

This expression has a negative sign in front which requires explanation. We measure the changes in these two variables between two points along a demand curve, and since the demand curve is negatively sloped, it implies that the ratio will be a negative value. Some textbooks choose to report the PED as a negative value and maintain consistency with the demand relationship. However, other textbooks, including ours here, will choose an alternative convention and simply drop the negative sign. Under this convention, PED is always reported as a positive value. The negative sign in the expression above is there to cancel out the sign of the ratio since two negatives make a positive.

The Price Elasticity of Demand measures how sensitive demand is to changes in price. For example, if demand for coffee decreases by 10% when the price of coffee increases by 5%, then the price elasticity of demand = -(-10/5) = 2. This means that demand increases two times as much as the price of coffee changes, in percentage terms.

Alternatively, if demand for coffee were to increase by 5% after the price of coffee decreased by 10% then price elasticity of demand, PED = -(5/(-10)) = 0.5. This means that the demand for coffee changes by half as much as the price changes, in percentage terms.

Notice that PED is a unitless value because each percentage change is also unitless. Therefore we can use these to compare sensitivities of demand to price changes between a wide variety of different products all measured in different units.

Calculating PED along a Market Demand Curve

Information from a demand curve can be used to derive the PED for a product. The easiest method is to find the values for price and quantity demanded at two separate points on the demand curve, calculate the percentage changes for each variable in moving from one point to the other, and forming the ratio of the percentage changes as in the PED formula above. However, this method generates a problem that results in a seeming inconsistency. Namely, the PED derived in moving from a point A to a point B along a demand curve will be different from the PED derived in moving from point B to point A. There are two ways to resolve this inconsistency. One method, used in many introductory textbooks, is to apply what is known as the mid-point formula. Although this method works well it is actually a bit more complicated than using the second method which uses a point-formula derived using calculus. We'll use the point formula going forward.

Side-Note: One of the purposes of calculus is to derive point-estimates of changes, rather than changes between two distinct points. The method is to assume that the two points are infinitesimally close to each other (often said to be within an epsilon). The first derivative of an equation gives us the result of that infinitesimal change and corresponds to the slope of a tangent line drawn through a particular point on a curve. However, when the curve is linear (a straight line), the first derivative is the slope of the line. Below we present the formula for the point estimate for PED that is derived using calculus, but you don't need to know calculus to use the formula and apply it.

Point-Formula for the Price Elasticity of Demand

The point-formula for the PED can be used whenever one knows the values of price and quantity on a demand curve and the slope of the demand curve at that point. The result is a unitless expression that has the same interpretation as the PED formula presented above.

Begin with the PED formula from above.

$$PED = - (\Delta Q_D/Q_D)*(P/\Delta P)$$

Then rearrange to get,

$$- (\Delta Q_D*P)/(\Delta P*Q_D)$$

And rearrange again to get,

$$- (\Delta Q_D/\Delta P)*(P/Q_D)$$

Now, since the slope of a demand curve can be written as $(\Delta P/\Delta Q_D)$ then,

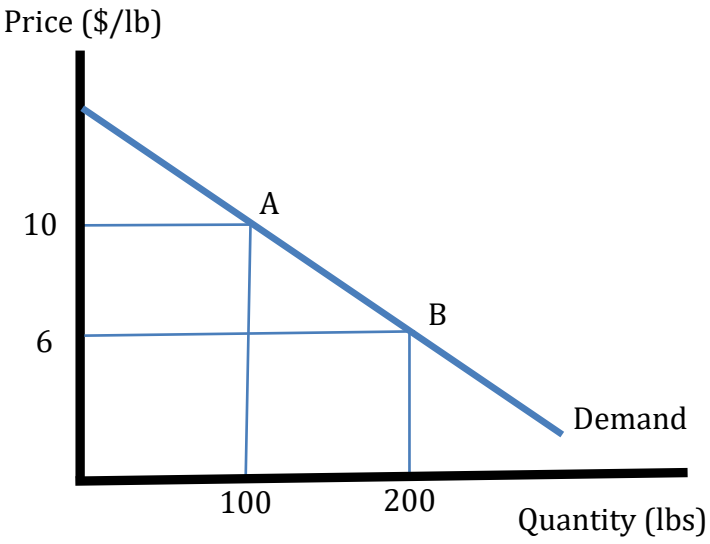
$$(\Delta Q_D/\Delta P) = 1/(\Delta P/\Delta Q_D) \text{ or } (1/\text{slope of demand curve})$$

Therefore,

$$PED = -\left(1/(\Delta P/\Delta Q_D)\right) * (P/Q_D) = -(1/\text{slope of demand curve}) * (P/Q_D)$$

Consider the linear demand curve in Figure 8.1. We can calculate the PED at each of the two points labeled A and B.

Figure 8.1 Calculating Elasticities on a Linear Demand curve



PED at Point A = $-(1/\text{slope of line}) \times (P/Q_D)$

$= -(1/(-4/100)) \times (10/100) = (100/4) \times (10/100) = 10/4 = +2.5$

PED at Point B = $-(1/\text{slope of line}) \times (P/Q_D)$

$= -(1/(-4/100)) \times (6/200) = (100/4) \times (6/200) = 6/8 = \frac{3}{4} = +0.75$

Notice that the PED is not the same at two different point on the same demand curve. There is more about that below but first we must introduce some descriptive terminology.

Economists use the following terms to describe PEDs that take different values.

If PED > 1 say that demand is ELASTIC

If PED < 1 say that demand is INELASTIC

If PED = 1 say demand is UNIT ELASTIC

If PED = 0 say that demand is PERFECTLY INELASTIC

IF PED = ∞ say that demand is PERFECTLY ELASTIC

Interpretations of PED

The price elasticity of demand is a measure of the sensitivity of market demand to changes in the price of the product. In general, when a good is elastic, it means that demand is very sensitive to changes in price, because demand is affected by a greater percentage than the price change itself (PED>1). When a good is inelastic, it means that demand is not very sensitive to changes in price, because demand is affected by a lesser percentage than the price change itself (PED<1).

Products that are elastic are often those that have many near substitutes. For example soda will likely be elastic since there are many other types of drinks one can choose from. In contrast, gasoline will likely be inelastic because of the large number of gasoline powered motor vehicles currently in use and the inability to substitute for other fuels.

The elasticity of a product may change over time though. If the price of gasoline remains high for a long period of time, more consumer may choose to purchase electric vehicles which would enable a substitution away from gasoline eventually. That means that gasoline may be inelastic in the short-run but more elastic in the longer term.

The elasticity of a product will also depend on the level of aggregation. For example, cheddar cheese is likely to be very elastic because there are many other types of cheese one can substitute. A more aggregated product such as cheese, will be less elastic because there are fewer substitutes for cheese. The even more aggregated product, food, will be even less elastic, or equivalently more inelastic, because there are no substitutes at that level of aggregation.

Finally, whether a good is a necessity or a luxury product will influence its elasticity. A necessity has very few substitutes, especially at a high level of aggregation, such as food, and thus would be inelastic. Luxuries are defined as products that may enhance the quality of one's life but which are not absolutely necessary to survive. Luxuries are more likely to be very elastic.

PED and the Demand Curve

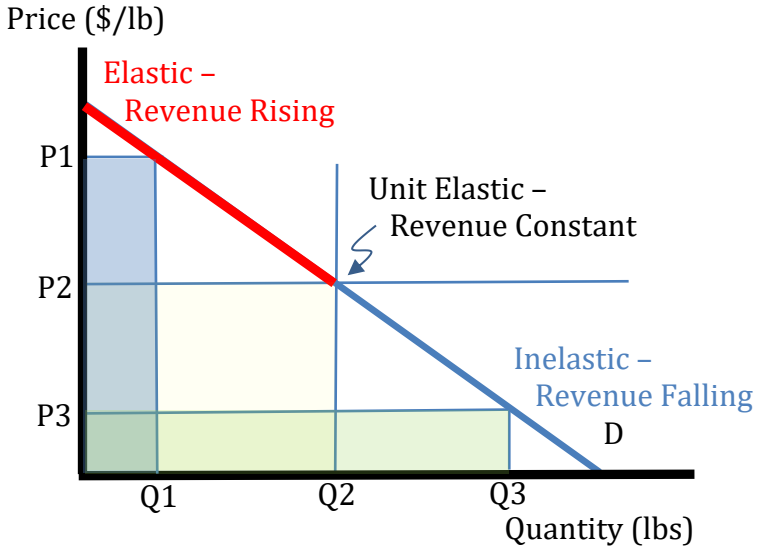
There are several regularities worth noting about the price elasticity of demand along a linear demand curve as shown in Figure 8.2. First, let's define Total Market Revenue, or just total revenue (TR), as the total amount of money earned by firms in the market when Q units are sold at a price P.

$$TR = P \times Q$$

In Figure 8.2 total revenue is depicted as an area on the diagram. For example, if the quantity Q1 were sold at the price P1, total revenue (TR1 = P1 x Q1) is depicted as the area of the bluish rectangle. Since price is measured in $/lb and quantity is measured in lbs, then

the product of the two (TR) is measured in $. (*Take note that all areas on a supply and demand diagram are always measured in monetary units, such as dollars*)

Figure 8.2 Elasticities and Revenue with a Linear Demand Curve



Likewise, the total revenue earned by firms at the quantity Q2 and price P2 is depicted as the yellowish area (TR2), and the total revenue earned by firms at the quantity Q3 and price P3 is depicted as the greenish area (TR3). These statements are true regardless of the curvature of the demand curve. However, for a linear demand curve there are some further regularities to note.

First, notice that the yellowish area, TR2, appears larger than the other two areas. This is true because Q2 is purposely chosen as the midpoint of the linear demand curve. Using calculus tools, it is possible to show that total revenue along a linear demand curve will be at its maximum value at the midpoint of the demand curve. That implies that in Figure 8.2, TR2 > TR1, and TR2 > TR3. It also means that as quantity increases from zero to the midpoint (along the red segment of the demand curve), total revenue increases until the maximum revenue is reached at the midpoint. As quantity increases past the midpoint Q2, total revenue decreases.

Next note that as the quantity increases along the demand curve, the price falls. Therefore, if the quantity were increasing at a faster percentage than the price was falling, then total revenue would have to be increasing as it is along the red segment. But if the percentage change in quantity were greater than the percentage change in price, the elasticity measure would have to be greater than 1, or elastic. In other words, along the red segment, (left of the midpoint) not only is total revenue rising with greater quantity, demand is also elastic. Thus, when demand is elastic, total revenue is rising as quantity increases and the price decreases.

The same is true along the lower blue segment of the demand curve. In that range, price is falling by a lower percentage than the quantity is rising and so demand is inelastic. Thus,

when demand is inelastic, total revenue is falling as quantity increases and the price decreases.

At the midpoint of a linear demand curve, the product is unit-elastic.

In summary, the price elasticity of demand measures the sensitivity of demand for a product in response to changes in the price. A product with elastic demand is associated with a relatively flat demand curve because small changes in price induce large changes in demand. A product with inelastic demand is associated with a relatively steep demand curve because large changes in price induce a small change in demand.

Income Elasticity of Demand (IED)

The income elasticity of demand measures how much demand for a product changes given a change in an individual's income. It is constructed in the same way as price elasticity, but substituting the percentage change in income for the change in price. But whose income should be used here? As with the market demand curve, income represents some measure of the community of consumers who potentially demand the product. So, it's best to think of this as community income.

$$\text{IED} = \%\text{ change in demand} / \%\text{ change in income} = -(\Delta Q_D/Q_D) / (\Delta I/I)$$

Consider a few examples. If demand increases by 1% when income increases 5% then the income elasticity of demand = (1/5) = 0.20.

Alternatively, if demand decreases by 20% when income increases by 10% then income elasticity of demand = - 20/10 = - 2.0.

Notice from the examples that IED may be positive or negative in value depending on whether the good is a normal or inferior good.

If IED > 0 → the good is a normal good

If IED < 0 → the good is an inferior good

Necessities are likely to have low income elasticities, close to zero, because people will tend to buy a similar amount of these goods even if income rises. Consider toothpaste. A wealthy person is likely to buy a similar amount as a poorer person. Thus, as income rises there is not much change in demand. However, luxury goods will tend to have higher income elasticities because as community income rises, more people will choose to purchase the products. Sometimes there is an income threshold, which once passed, demand rises quickly. For example, consider the demand for automobiles in a developing economy once incomes reach something close to middle class levels. Many households begin to purchase cars and demand rises swiftly.

Note: one cannot derive the IED for a good by knowing values on its demand curve. This is because the demand curve is not plotted in income-space and therefore the relevant info is not present.

Cross Price Elasticity of Demand (CPED)

The cross-price elasticity of demand measures how much demand for a product changes given a change in the price of another good. It is constructed in the same way as price elasticity, but substituting the percentage change in a different product for the change in own-price. But which other prices are relevant here? As shown in the market demand curve, the prices of substitute and complement goods affect market demand so these are the other prices that matter.

CPED = % change in demand / % change in another product's price

or,

$$CPEDx = -(\Delta Q_x/Q_x) / (\Delta Py/Py)$$

Where Qx is the product demanded and Py is the price of a different good.

Consider a few examples. If demand for coffee increases by 10% after the price of tea increases by 20% then the cross price elasticity of demand = 10/20 = +0.5. Alternatively, if demand for coffee increases by 10% when the price of sugar decreases by 20% then the cross price elasticity of demand = 10/-20 = -0.5.

Notice from the examples that CPED may be positive or negative in value depending on whether the other good is a substitute or a complement.

If CPED > 0 → the other good is a substitute

If CPED < 0 → the other good is a complement

Key Takeaways

1. The price elasticity of demand measures the sensitivity of demand to changes in the price in percentage terms.
2. The income elasticity of demand measures the sensitivity of demand to changes in the community income in percentage terms.
3. The cross-price elasticity of demand measures the sensitivity of demand of good x to changes in the price of good y in percentage terms.
4. A flat (steep) demand curve represents a product with elastic (inelastic) demand.
5. When demand is linear and elastic, total revenue increases as the quantity increases and as the price decreases.
6. When demand is linear and inelastic, total revenue decreases as the quantity increases and as the price decreases.
7. A normal (inferior) good has a positive (negative) income elasticity of demand.
8. A positive (negative) cross-price elasticity of demand indicates that the two goods being compared are substitutes (complements).

8.2 Consumer Surplus

Learning Objectives

1. Learn how to measure the total welfare of consumers purchasing a product in a market.
2. Learn how to measure the change in consumer surplus caused by a change in the price.

Consumer surplus is a very important concept that will be used throughout the rest of the text in the evaluation of market changes and the effects of government policies. One of the questions we will ask over and over again is, are market participants made better-off or worse-off as a result of some change? We care about this because we seek to identify market arrangements that will enhance the welfare of individuals and households in a community.

Consumer surplus is a measure of the welfare of the consumers who purchase a particular product in a market. It uses information that is depicted on a market demand curve. Note that the term welfare has different meanings in different contexts. Here we use welfare as synonymous with well-being and are not associating this term with government welfare programs.

We introduced the term surplus in Chapter 3 to described the extra value, or happiness, that accrues to Smith and Jones when they trade oranges for apples. Recall, that mutually voluntary trade occurs only if both individuals expect to realize an improvement in their utility. This was represented diagrammatically as a movement onto a higher indifference curve for each of them. Surplus in that context is the increase in utility that results from trade and is measured in utils, which is the fictitious unit of happiness measurement that we use because we don't know how to measure happiness directly. The inability to measure something this important is a problem that will be solved by introducing the surplus measure defined here, because consumer surplus will be measured in monetary units, or dollars. In addition, consumer surplus in the market will account for the improvement in welfare not of just one individual, but of all the individuals who purchase the product at the market price. Nevertheless, the concept of consumer surplus is exactly the same as defined previously, except that it is measured differently and is more comprehensive.

Consumer surplus is found by thinking very hard about what a demand curve tells us. Consider the demand curve in Figure 8.3 with some very simple numerical values. Recall the a demand curve is hypothetical in the sense that it tells us what demand would be at different prices that could prevail. It does not tell us what the price is directly. So, imagine that the price in the market for this product is P_{Mkt} = \$1 per pound and that every person can purchase the product at that same price. From the diagram, total market demand would be 5 lbs of the product.

Next, let's make the simplifying assumption that each one of the five lbs is purchased by a different individual. (We make this assumption just so the story is easier to tell). Finally, assume that three of the people who purchase a pound of the good are named Bi, Bop and Bo.

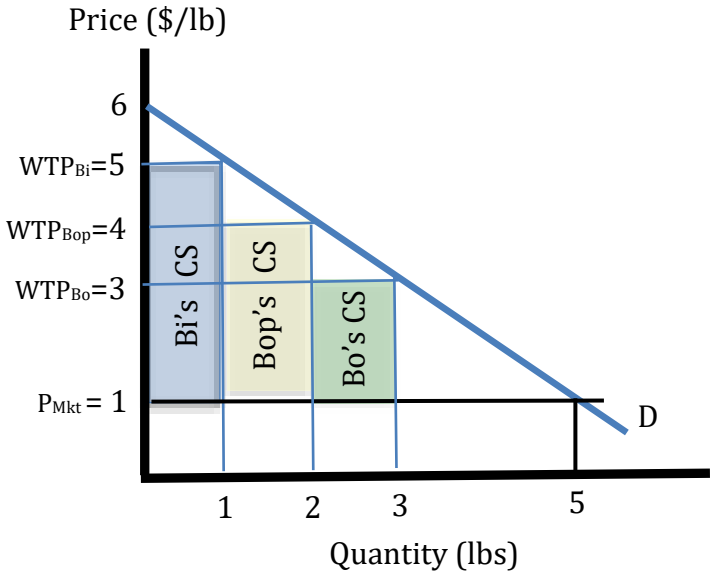
Next let's consider a hypothetical alternative scenario. If the market price were \$5 rather than \$1, then market demand would be only one pound and that unit would be purchased by Bi. Another way to state this outcome is that Bi is "willing to pay" \$5 to purchase a pound of the product. Bop and Bo are not willing to pay that much and so would stay out of the market.

Next let's compare this outcome for Bi with the original assumption that the market price is $1. The alternate scenario suggests that Bi is willing to pay as much as $5 ($WTP_{Bi}$ - $5) but actually pays only $1. The difference in prices ($4) is what we'll use to measure Bi's surplus value attained at the market price of $1. It is the bluish area on the graph with height $4 per pound and width of 1 pound. This surplus measures the extra value, or happiness, that Bi receives when he purchases the product at the market price of $1.

Next, we can note hypothetically that Bop would enter the market and purchase the second pound of the good if the price were $4 ($WTP_{Bop}$ = $4). However, since the market price is only $1 in the first scenario, he receives a surplus of $3 when purchasing the second pound. This is given by the yellowish area on the graph.

Finally, Bo would be willing to pay at most $3 ($WTP_{Bo}$ = $3), but since he only pays $1 he attains a surplus of $2 (the greenish area) on the third pound sold.

Figure 8.3 Individual Consumer Surplus

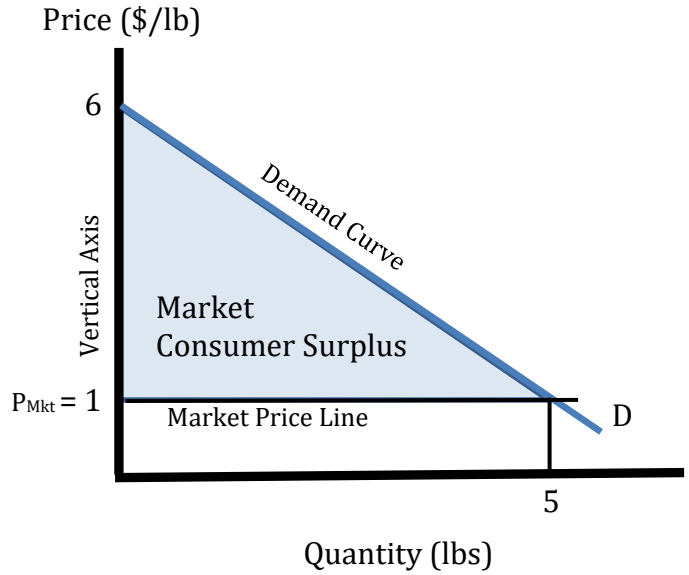


At this stage in the story, the total consumer surplus in the market is the sum of the surpluses of the three consumers, $4 + $3 + $2 = $9. But of course there would be a little more. A fourth consumer would clearly be willing to pay $2 to purchase the fourth pound, etc, etc. One awkward feature of this result is the small white triangles that lie above each individuals surplus rectangle which would not be a part of the consumer surplus. However, these triangles can be eliminated with a simple assumption change. Suppose that the units listed are not in pounds, but rather millions of pounds. In this case, one unit would be a very small distance along the quantity axis and Bi's surplus rectangle would be extremely thin and reach all the way up to the demand curve at the price of $6. The small triangles would all become subsumed in the individual surpluses of the, now millions, of consumers. With this procedure in mind we can now derive a simple estimate for market consumer surplus as the area between the demand curve, the horizontal line drawn at the market price and the vertical axis. This is shown clearly in Figure 8.4 as the blue area. Reverting back to the original values on the graph, we can easily calculate consumer surplus in the market as the area of the triangle.

$$CS = \tfrac{1}{2}(6 - 1)(5) = \$12.5$$

Again, note that $12.50 represents the total amount of extra money that consumers in this market would have been willing to pay in different circumstances, but didn't actually have to pay because they were each charged $1 per pound. It corresponds to the extra utility attained by the consumers in the market because of trade. As such, it is a measure of the extra welfare accruing to the consumers of this product relative to the situation where no trading took place.

Figure 8.4 Measuring Market Consumer Surplus



Reality Check

At this stage, some may begin to feel that concepts like consumer surplus are easy to identify on a graph and to measure, but that the concept is very abstract and difficult to relate to. One might ask if people actually feel this surplus value in the real world. The answer is yes.

It's easier to understand, if we imagine a purchase that is deemed so good that the buyer wants to tell everyone about the great deal he just got. For example, imagine someone who always wanted to own a vintage Les Paul model Gibson guitar like some of the great rock musicians of the past. However, with prices starting at $2500 it takes a long time to save up the money to purchase one. Suppose this person eventually saves $5000 to buy a really good guitar but once he begins searching for it, he comes across one in prime condition at an estate sale with an asking price of just $1000. It's the kind of deal he would want to tell all his like-minded friends and enthusiasts about. That happiness he feels when bringing that guitar home is measured at $4000 and is his consumer surplus for that purchase. Probably all of us have had that kind of feeling at least a few times before.

But what about the purchase of toothpaste at the drug store? Does that really generate surplus value for the purchaser? Again the answer is yes, only in this case it is not so pronounced. The fact that someone is willing to pay $2.50 for a tube of toothpaste is done because the value received is greater than the money given up, even if only by a few cents. Still it is considered surplus value and it is a part of the surplus accruing in small and large amounts every time a transaction is completed.

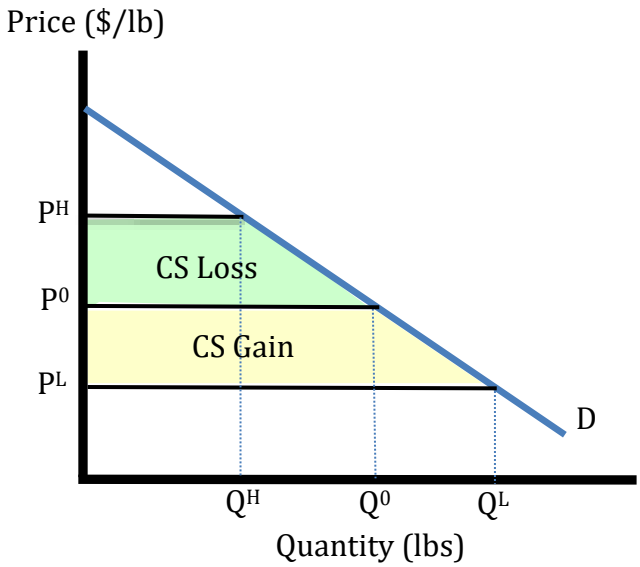
You "feel," or notice, consumer surplus whenever you buy something of high value to you at a very low price and then tell all your friends and family about the great "deal." That feeling is consumer surplus.

Changes in Consumer Surplus

Later we will apply the concept of consumer surplus to measure the well-being of consumers is various market outcomes. Often we will ask not only what is the level of consumer surplus in a particular equilibrium, but also how does consumer surplus change when there is a change in the market prices and quantities. This distinction is important enough to make a mental note now for future reference. For questions of problems you face in this course assessing consumer surplus, always ask whether it is asking for the **level** of consumer surplus, or the **change in** consumer surplus. In the Figure 8.4, we measured the level of consumer surplus. In Figure 8.5 we note how to measure the change in consumer surplus.

Consumer surplus changes whenever there is a change in the market price and/or the quantity that consumers purchase. Generally, these two variables will both change together.
In Figure 8.5, assume that the initial price in the market is P^o and the quantity consumed is Q^o. Suppose further that the price in the market falls, for some unspecified reason, to the lower price P^L and the quantity purchased rises to Q^L. In this case the change in consumer surplus (ΔCS) will be positive and measured by the yellowish area labeled CS Gain. The increase in surplus arises for two reasons. First, all of the consumers who purchased the product at the price Po are still purchasing the product afterwards, but at a lower price. These consumers enjoy an increase in surplus given by the area $(P^o - P^L)Q^o$. Second, there are additional consumers who enter the market because the price has fallen. They enjoy surplus that they didn't get previously, given by the area $\frac{1}{2}(P^o - P^L)(Q^L - Q^L)$.

Figure 8.5 Changes in Consumer Surplus



Next consider another scenario in which the price of the product rises, for some unspecified reason, to the price P^H and the quantity purchased falls to Q^H. In this case the change in consumer surplus (ΔCS) will be negative and measured by the greenish area labeled CS Loss.

The decrease in surplus arises for two reasons. First, there are the consumers who purchased the product at the price Po and are still purchasing the product afterwards, but at a higher price. These consumers lose surplus given by the area $(P^H - P^o)Q^H$. Second, there are the consumers who drop out of the market because the price has increased. They lose surplus given by the area $\frac{1}{2}(P^H - P^o)(Q^H - Q^o)$.

In general, the change in consumer surplus is given by the area between the two prices (before and after) the demand curve and the vertical axis. If the price increases, consumer surplus falls because consumers dislike higher prices. If the price falls, consumer surplus rises because consumers like lower prices.

Finally, it is common to ask who loses from, say, an increase in the price of butter. A typical response is that consumers lose. However, it is important to be more careful with language. Everyone in the country is a consumer, so when one says that consumers lose from an increase in the price of butter, that would seem to imply that everyone loses because everyone is a consumer. This inference is wrong though. Only the consumers of butter, or potential consumers of butter, lose from the increase in the price, not all consumers. Any individual who would never purchase butter, regardless of the price, is a consumer who is unaffected by the price change. Thus if a question asks, *specifically*, who loses from an increase in the price of butter, it is correct to say *consumers of butter* lose. Precision in language helps to prevent misunderstandings.

Key Takeaways

1. Consumer surplus measures the total amount of welfare, or well-being, that accrues to consumers who purchase a product in a market.
2. Consumer surplus is the area between the demand curve, the horizontal line drawn at the market price and the vertical axis.
3. The change in consumer surplus is given by the area between the two prices (before and after) the demand curve and the vertical axis.
4. Specifically, the consumers affected by a price change are only those consumers in the market for that product.